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TIME-TO-GO PREDICTION FOR A HOMING MISSILE BASED ON MINIMUM-TIME TRAJECTORIES[1]

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Abstract

In two dimensions, the trajectory of a constant velocity missile which intercepts a zero-acceleration target and which minimizes the weighted sum of the final time and the integral of the missile normal acceleration squared is derived. The launch angle of the missile is arbitrarily prescribed. The optimal control involves elliptic functions and requires the numerical solution of two non-linear algebraic equations for its calculation. This procedure is used to calculate the time-to-go in a six-degree-of-freedom simulation in which the target performs accelerated maneuvers. The current missile velocity is projected on the plane of the line of sight vector and the target vector, and the time required to perform the two-dimensional intercept is calculated and used as the time-to-go for the linear-quadratic guidance law of the missile. Results show substantial improvement relative to the range-over-closing-speed method of computing time-to-go.

List of Symbols

a_n	Missile normal acceleration (ft/sec^2)
A, B, C	Constants defined in Eqs. (21) through (23)
G	End-point function
H	Hamiltonian
J	Performance index
K	Modulus of elliptic functions
P	Magnitude of missile acceleration oscillation
t	Time (sec)
v	Ratio of target speed to missile speed
V	Velocity (ft/sec)
W	Weight in the performance index
X, Y	Inertial coordinates
α_n	Nondimensional missile normal acceleration
γ	Angle of maximum normal acceleration
ξ	Nondimensional relative abscissa
η	Nondimensional relative ordinate
θ	Orientation of missile velocity vector to X-axis
λ	Variable Lagrange multiplier
τ	Nondimensional time
φ	Orientation of target velocity to X-axis
ψ	Transformed variable representing θ

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Introduction

A guidance law of current interest for bank-to-turn homing missiles is the linear-quadratic guidance law which contains proportional navigation as a particular case (see, for example, Ref. 1). In order to implement this guidance law, an algorithm for predicting time-to-to is needed. The simplest time-to-go formula is range divided by closing speed and is valid for a constant-velocity missile and target on a collision course. This formula has been improved in Ref. 2 by accounting for the missile longitudinal acceleration.

Unfortunately, the linear-quadratic guidance law tends to drive the missile and the target into a homing triangle in which range and closing speed become unobservable. In Ref. 1, a linear-quadratic guidance law for dual control (intercept and estimation enhancement) has been proposed. This guidance rule moves the missile away from the homing triangle improving estimation but making the time-to-go algorithm invalid.

The purpose of this study is to develop a time-to-go algorithm which is suitable for dual control. This is accomplished by finding the normal acceleration history which minimizes the time for a constant-velocity missile to intercept a nonaccelerating target and using this time as the time-to-go. To account for a limit on the normal acceleration, the performance index is weighted by the integral of the control squared.

As a first step, the analysis is carried out in two dimensions with the hope that some insight in the three-dimensional problem will be achieved. The resulting time-to-go algorithm is tested in a six-degree-of-freedom simulation by projecting the current missile velocity vector onto the plane of the line of sight and the target velocity. The optimal intercept time is computed in this plane and used as the time-to-go for the linear-quadratic guidance law. Missile acceleration is included by using an average missile speed.

Optimal Intercept Problem

The geometry of the intercept problem is shown in Fig. 1. The XY coordinate system represents an inertial frame, and the X axis is along the line of sight at $t = 0$. The target, located at $X_T = X_o$ at $t = 0$, is moving along a straight line which makes an angle ϕ with respect to the X axis. The constant-speed missile is launched at an angle θ_o relative to the X axis, and the velocity direction $\theta(t)$ is changed by controlling the normal acceleration $a_n(t)$.

In terms of the nondimensional quantities

$$\tau = t\frac{V_M}{X_o}, \quad \xi = \frac{X_T - X_M}{X_o}, \quad \eta = \frac{Y_T - Y_M}{X_o}$$
$$\text{(1)}$$
$$\alpha_n = \frac{a_n X_o}{V_M^2}, \quad v = \frac{V_T}{V_M},$$

the optimal intercept problem is stated as follows: Find the normal acceleration history $\alpha_n(\tau)$ which minimizes the performance index

$$J = W(\tau_f - \tau_o) + \frac{1-W}{2}\int_{\tau_o}^{\tau_f}\alpha_n^2 d\tau \quad \text{(2)}$$

subject to the differential constraints

$$\dot{\xi} = v\cos\phi - \cos\theta \quad \text{(3)}$$
$$\dot{\eta} = v\sin\phi - \sin\theta \quad \text{(4)}$$
$$\dot{\theta} = \alpha_n \quad \text{(5)}$$

and the prescribed boundary conditions

$$\tau_o = 0, \quad \xi_o = 1, \quad \eta_o = 0, \quad \theta_o = \theta_{o_S} \quad \text{(6)}$$
$$\xi_f = 0, \quad \eta_f = 0 \quad \text{(7)}$$

where θ_{o_S} is the specified value of θ_o.

The performance index is the weighted sum of the final time and the integral of the normal acceleration squared. Without the integral, the solution of the optimal control problem is infinite normal acceleration to rotate the missile velocity to the homing triangle direction followed by zero normal acceleration until intercept. The physical bound which exists on normal acceleration can be incorporated directly as a control variable inequality constraint. This approach makes the optimal control be maximum normal acceleration followed by zero normal acceleration and is the subject of a companion study (Ref. 3). Here, the normal acceleration bound is included indirectly by imposing a penalty on the performance index for high normal acceleration. By adjusting the weight W, the highest normal acceleration encountered along the trajectory can be controlled.

Solution of the Optimal Control Problem

From the variational Hamiltonian

$$H = \frac{1-W}{2}\alpha_n^2 + \lambda_\xi(v\cos\phi - \cos\theta)$$
$$\text{(8)}$$
$$+\lambda_\eta(v\sin\phi - \sin\theta) + \lambda_\theta(\alpha_n),$$

it is seen that the Euler-Lagrange equations are given by

$$\dot{\lambda}_\xi = -H_\xi = 0 \quad \text{(9)}$$
$$\dot{\lambda}_\eta = -H_\eta = 0 \quad \text{(10)}$$
$$\dot{\lambda}_\theta = -H_\theta = -\lambda_\xi\sin\theta + \lambda_\eta\cos\theta \quad \text{(11)}$$
$$0 = H_{\alpha_n} = (1-W)\alpha_n + \lambda_\theta \quad \text{(12)}$$

Eqs. (9) and (10) state that λ_ξ and λ_η must be constants. Before solving the remaining equations, the natural boundary conditions are derived.

From the end-point function

$$G = W(\tau_f - \tau_o) + \nu_\xi\xi_f + \nu_\eta\eta_f, \quad \text{(13)}$$

it is seen that the natural boundary conditions are

$$H_f = -G_{\tau_f} = -W \quad \text{(14)}$$
$$\lambda_{\xi f} = G_{\xi_f} = \nu_\xi \quad \text{(15)}$$
$$\lambda_{\eta f} = G_{\eta_f} = \nu_\eta \quad \text{(16)}$$
$$\lambda_{\theta f} = G_{\theta_f} = 0 \quad \text{(17)}$$

Eqs. (12) and (17) imply that

$$\alpha_{nf} = 0 \quad \text{(18)}$$

Since the Hamiltonian is not an explicit function of time, the first integral $H = $ const exists. When combined with Eq. (14), the first integral becomes

$$H = -W \quad \text{(19)}$$

Hence, if the definition (8) of H is substituted into Eq. (19), the following expression for the optimal control can be obtained

$$\alpha_n = \pm\sqrt{\frac{2}{1-W}(A + B\sin\theta + C\cos\theta)} \quad \text{(20)}$$

where

$$A = W + \lambda_\xi v\cos\phi + \lambda_\eta v\sin\phi \quad \text{(21)}$$
$$B = -\lambda_\eta \quad \text{(22)}$$
$$C = -\lambda_\xi \quad \text{(23)}$$

If θ_o were free, the optimal α_n would be $\alpha_n = 0$, and the optimal intercept path would be $\theta = $ const. $\equiv \theta_{SL}$ or a straight line. Hence, if $\theta_o < \theta_{SL}$, the plus sign in Eq. (20) holds, and if $\theta_o > \theta_{SL}$, the minus sign is valid.

Eq. (20) can also be obtained by integrating Eq. (11). This is accomplished by changing the independent variable from τ to θ using Eq. (5), using Eq. (12) to relate α_n to λ_θ, integrating, and applying the final condition (17). Finally, θ_f is eliminated by using the final condition (14).

In order to integrate the differential equations (3) through (5), it is useful to rewrite Eq. (20) in the elliptic function form (Ref. 4)

$$\alpha_n = \pm\sqrt{\frac{2}{1-W}(A+P)(1 - K^2\sin^2\psi)} \quad \text{(24)}$$

where

$$\psi = \frac{\theta - \gamma}{2}, \quad \tan\gamma = \frac{B}{C}, \quad P = \sqrt{B^2 + C^2}, K^2 = \frac{2P}{A+P} \quad \text{(25)}$$

If τ is chosen to be the dependent variable, Eq. (5) can be integrated as

$$\tau_{go} = \tau_f - \tau_o = \int_{\theta_o}^{\theta_f}\frac{d\theta}{\alpha_n(\theta)} = 2\int_{\psi_o}^{\psi_f}\frac{d\psi}{\alpha_n(\psi)} \quad \text{(26)}$$

Then, substitution of Eq. (24) leads to

$$\tau_{go} = \pm K\sqrt{\frac{1-W}{P}}[F(\psi_f, K) - F(\psi_o, K)] \quad \text{(27)}$$

where $F(\psi, K)$ is the elliptic integral of the first kind.

Similarly, Eqs. (3) and (4) can be expressed as

$$\xi - \xi_o = \pm 2 \int_{\psi_o}^{\psi} \frac{[v \cos \phi - \cos(2\psi + \gamma)]d\psi}{\alpha_n(\psi)} \tag{28}$$

$$\eta - \eta_o = \pm 2 \int_{\psi_o}^{\psi} \frac{[v \sin \phi - \sin(2\psi + \gamma)]d\psi}{\alpha_n(\psi)}. \tag{29}$$

Carrying out the integration and applying the initial conditions (6) leads to

$$\xi = \pm K \sqrt{\tfrac{1-W}{P}} \{[v \cos \phi - (1 - \tfrac{2}{K^2}) \cos \gamma][F(\psi, K) - F(\psi_o, K)]$$
$$\qquad - \tfrac{2 \cos \gamma}{K^2}[E(\psi, K) - E(\psi_o, K)] \tag{30}$$
$$\qquad - \tfrac{\sin \gamma}{K^2}[\sqrt{1 - K^2 \sin^2 \psi} - \sqrt{1 - K^2 \sin^2 \psi_o}]\} + 1$$

and

$$\eta = \pm K \sqrt{\tfrac{1-W}{P}} \{[v \sin \phi - (1 - \tfrac{2}{K^2}) \sin \gamma][F(\psi, K) - F(\psi_o, K)]$$
$$\qquad - \tfrac{2 \sin \gamma}{K^2}[E(\psi, K) - E(\psi_o, K)] \tag{31}$$
$$\qquad + \tfrac{\cos \gamma}{K^2}[\sqrt{1 - K^2 \sin^2 \psi} - \sqrt{1 - K^2 \sin^2 \psi_o}]\}$$

where $E(\psi, K)$ is the elliptic integral of the second kind.

The last step is to satisfy the final conditions (7). This amounts to finding the values of λ_ξ and λ_η which satisfy Eqs. (30) and (31) evaluated at ψ_f where $\xi_f = \eta_f = 0$. Because these equations are nonlinear, the solution process is iterative. Rather than solving for the Lagrange multipliers, it is more convenient to work in terms of ψ_o and ψ_f. The relationships between the ψ's and the λ's are obtained from Eqs. (24) and are given by

$$\lambda_\xi = \frac{-W \cos(\theta_o - 2\psi_o)}{2 \sin^2 \psi_f - 1 + v \cos(\phi - \theta_o + 2\psi_o)} \tag{32}$$

$$\lambda_\eta = \frac{-W \sin(\theta_o - 2\psi_o)}{2 \sin^2 \psi_f - 1 + v \cos(\phi - \theta_o + 2\psi_o)} \tag{33}$$

Numerical Results

Given values for W, v, ϕ, and θ_o, the values of ψ_o and ψ_f which satisfy Eqs. (30) and (31) evaluated at the final point are computed using Newton's method with analytical derivatives. Given ψ_o and ψ_f, λ_ξ and λ_η follow from Eqs. (32) and (33). Next, Eqs. (21) through (23) give A, B, and C, and Eqs. (25) give γ, P, and K. Finally, the values of ξ_f and η_f can be computed from Eqs. (30) and (31). Improved values for ψ_o and ψ_f (values which make ξ_f and η_f closer to zero) are provided by Newton's method.

Numerical results have been generated for the following parameter values:

$$v = 0,\ 0.2,\ 0.4,\ 0.6,\ 0.8$$
$$\phi = 0°,\ 45°,\ 90°$$
$$-70° \leq \theta_o \leq 70°$$

The limits on θ_o are dictated by the field of view of the seeker. Results are available for ϕ up to 180° in Ref. 4, but they have not been presented here because of space limitations. After some experimentation, the value of the weight has been chosen to be $W = 0.4$. This value gives reasonable trajectories for all parameter values.

The figures presented here have been generated mainly to verify the computational procedure. Figs. 2 through 4 contain ψ_o and ψ_f. Next, the normal acceleration histories are shown in Figs. 5 through 7 for the case where $v = 0.5$ or $V_M = 2V_T$. Then, Figs. 8 through 10 present the flight time, and finally, the trajectories are presented in Figs. 11 through 13. The results presented in all the figures seem reasonable in that they follow expected trends.

Time-To-Go Prediction for an Accelerating Missile

It is desired to use the previous results in a six-degree-of-freedom homing missile simulation in which the missile velocity is not constant and the target acceleration is not zero. First, the missile velocity vector in the simulation is projected onto the plane of the line-of-sight vector and the target velocity vector, and the time-to-go is calculated in this plane. Second, a nominal tangential acceleration history is assumed for the missile and a constant average missile velocity is determined for a given initial time and an assumed final time. This velocity is then used to compute the time for intercept as defined in Section 3. Since the computed final time and the assumed final time are not equal, an iterative process is used to compute the final time and, hence, the current time-to-go.

Some results are presented in Table 1 for a target which performs two maximum acceleration maneuvers: one when the missile is 6,000 feet away and one when the missile is 1.0 sec away. Miss distances are shown for two off-boresight angles, two launch ranges, and seven aspect angles. In Case I, time-to-go is calculated as range over closing speed, and in Case II, it is calculated using the minimum-time solution. In general, the new time-to-go algorithm yields a smaller miss distance than range over closing speed. In several cases it gives a hit where the latter gives a miss (more than ten feet).

Discussion and Conclusions

A time-to-go algorithm has been developed which is valid for missile-target geometries which differ greatly from the intercept triangle. The algorithm is based on the analytical solution of a weighted minimum-time problem. While the algorithm has been developed for a constant velocity missile in two dimensions, it can be applied to an accelerating missile in three directions by using an average missile velocity and by computing the time-to-go in the plane of the line-of-sight vector and the target velocity vector. A six-degree-of-freedom simulation shows that the proposed time-to-go algorithm improves miss-distance performance relative to computing time-to-go as range divided by closing speed.

Acknowledgement

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Fig. 1 , Intercept Geometry

Table 1: Simulation Results

Off-Boresight Angle (deg)	Launch Range (ft)	Aspect-Angle (deg)	Case I Miss Distance (ft)	Case II Miss Distance (ft)
0	7000	0	0.156	0.337
0	7000	30	0.694	0.243
0	7000	60	1.203	0.284
0	7000	90	1.322	0.469
0	7000	120	2.561	0.265
0	7000	150	0.739	0.077
0	7000	180	0.351	0.010
0	3000	0	0.983	0.097
0	3000	30	0.703	0.069
0	3000	60	1.382	0.063
0	3000	90	2.512	0.138
0	3000	120	11.85	2.950
0	3000	150	4.166	2.911
0	3000	180	0.605	0.253
40	7000	0	301.7	1251.
40	7000	30	230.9	1450.
40	7000	60	0.156	0.970
40	7000	90	0.910	0.296
40	7000	120	1.947	0.092
40	7000	150	1.546	0.323
40	7000	180	0.376	0.063
40	3000	0	2116.	0.041
40	3000	30	2700.	0.238
40	3000	60	2811.	0.165
40	3000	90	2520.	0.267
40	3000	120	1878.	0.386
40	3000	150	693.6	12.55
40	3000	180	229.6	267.2

Fig. 2a, ψ_o for $\phi = 0$ deg

Fig. 2b, ψ_f for $\phi = 0$ deg
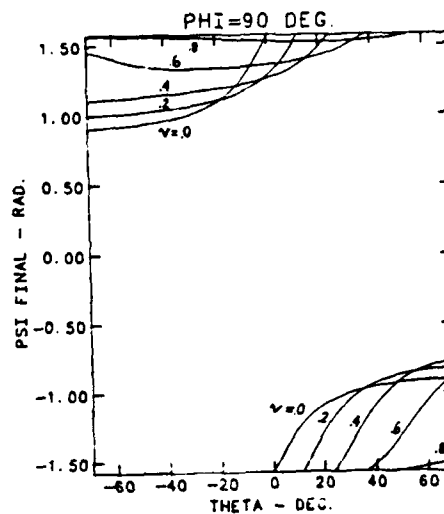
Fig. 3a,　ψ_o for $\phi = 45$ deg

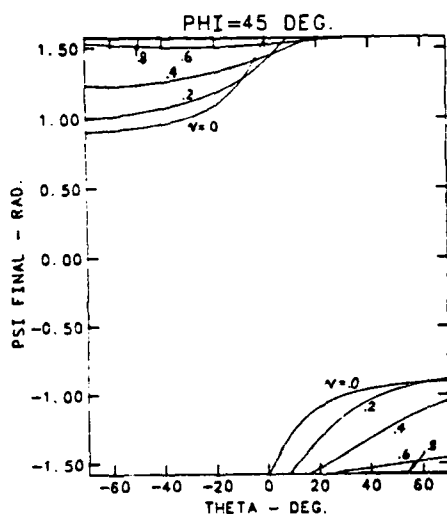
Fig. 4b,　ψ_f for $\phi = 90$ deg

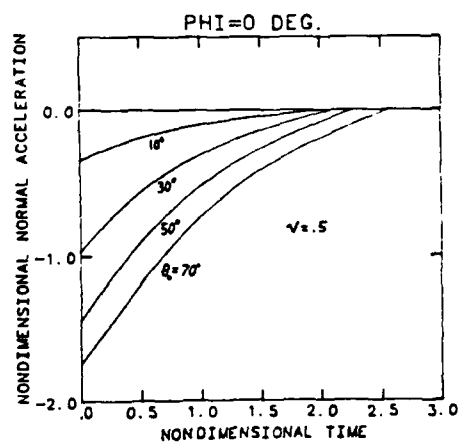Fig. 3b,　ψ_f for $\phi = 45$ deg
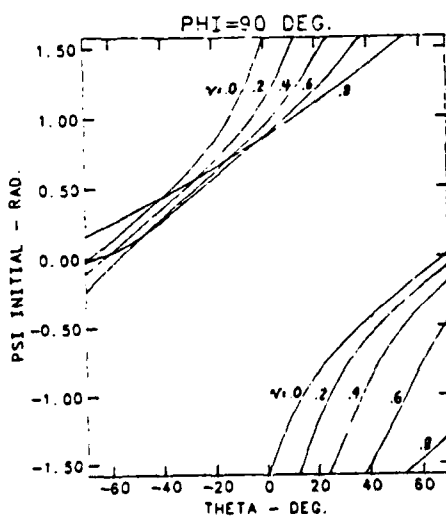

Fig. 5 ,　Control History, $\phi = 0$ deg
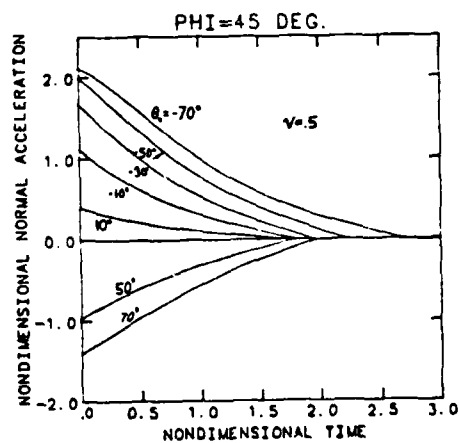

Fig. 4a,　ψ_o for $\phi = 90$ deg

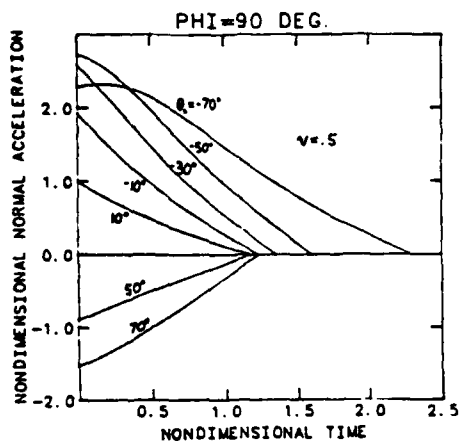Fig. 6 ,　Control History, $\phi = 45$ deg
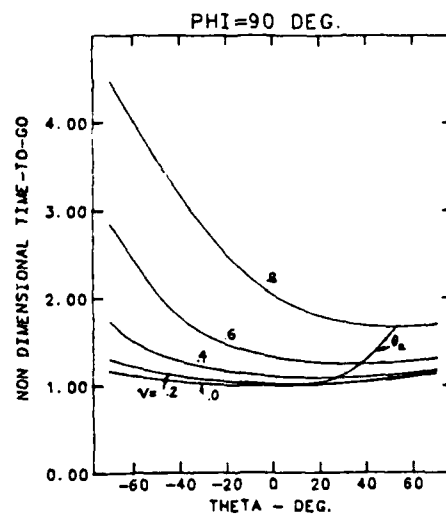
Fig. 7 , Control History, $\phi = 90$ deg
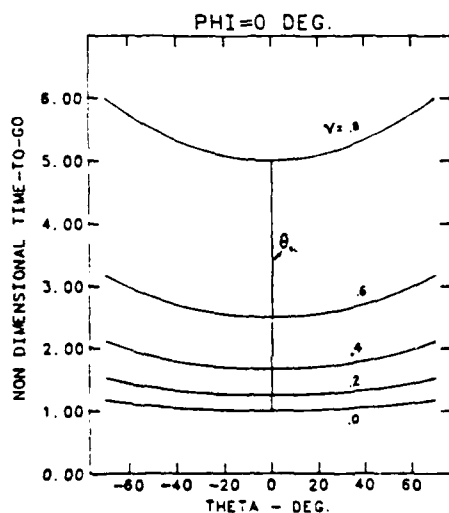

Fig. 10 , Time-to-Go, $\phi = 90$ deg

Fig. 8 , Time-to-Go, $\phi = 0$ deg

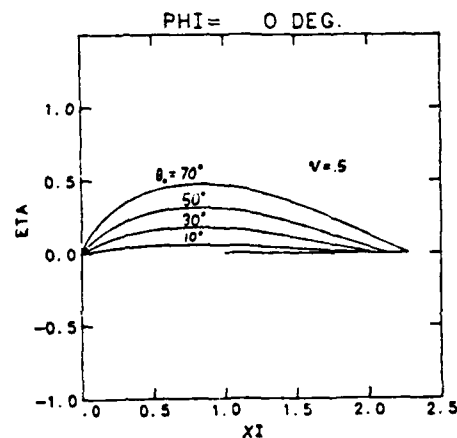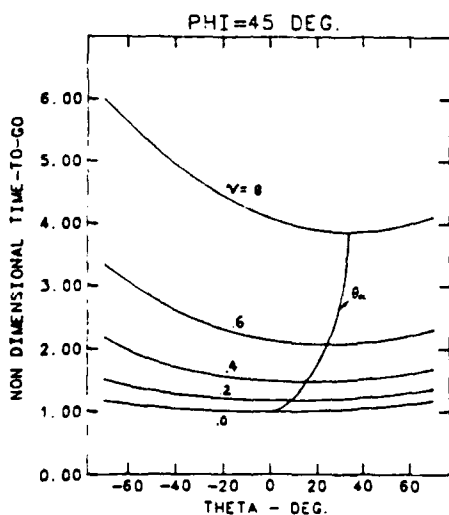Fig. 11 , Trajectories for $\varphi = 0$ deg
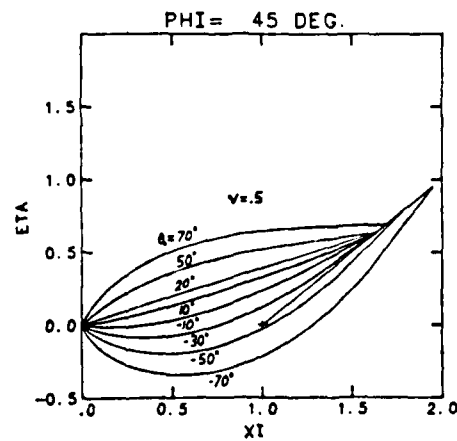
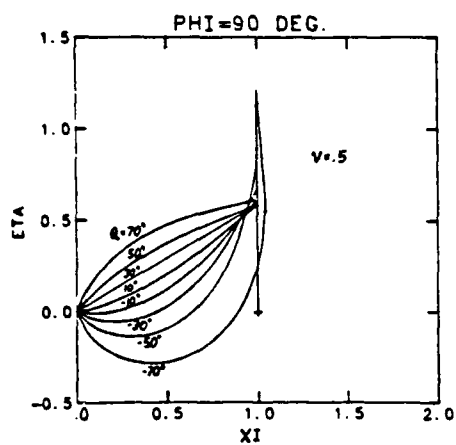

Fig. 9 , Time-to-Go, $\phi = 45$ deg

Fig. 12 , Trajectories for $\phi = 45$ deg

Fig. 13 ,   Trajectories for $\phi = 90$ deg